ANTHROPOLOGICAL AND MATHEMATICAL BASES OF COUNTING AND NUMBER SYSTEMS

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Quantification and Abstraction

We note the varying levels of abstraction in the history of quantification, counting, and mathematics. At first, there was a direct correspondence between items (especially body parts) and quantities; then, the rise of words unconnected to the physical objects. Next, "numbers" were developed as abstract concepts; and finally, mathematics came into being, with lots of abstraction between the quantities and symbols. There is now very little correlation between physical quantities and the figures which we shift around performing mathematics.

Counting systems

Basic counting systems started with biological bases. Most cultures used the digits of one hand, both hands, or hands and feet as a counting device. They might bring their fingers out from a fist, or vice−versa; this common method, still widely in use, allows for counting up to 10. With a "ones" hand and a "fives" hand, one can count up to 30. A variety of knuckle−counting methods can increase capacity, one able to represent figures up to 10 billion!

Many cultures developed body−counting systems, indicating a body part for each counted number (see Figure 1). These systems generally reach up to about 30, with about half on each symmetrical side of the body. In a couple of those systems, for example, the number 53 means "two men, and a third's middle toe" − two men with twenty digits, ten fingers and three toes on a third. In addition, complex gesture systems, depending on arm, hand, and finger position, were also used extensively, mostly in Europe (see Figure 2).

Object counting was another ancient technique. We have all heard the story about the shepherd who counted his sheep at the end of the day as they passed, dropping pebbles into a box; a more advanced shepherd would have boxes for different magnitudes, and thus use fewer pebbles. Notched pieces of wood known as tally sticks were widely used in the past as a reliable matching−tally device. The most ancient method was making notches in bones and sticks; this was done by Cro−Magnon man. Another approach is the use of knots for numeration, as the Incas did.

Number systems

Cultures all over the world came up with number systems whose bases varied widely.

Many of the most primitive societies only established words for "one" and "two," leaving themselves unable to count past about eight (it's impractical to keep track of the elements of a number like "two−two−two−two−two"). Other peoples, accustomed to counting with their fingers, used base 5 systems, and the names of the numbers frequently reflected their biological origin (for the Api people, 5 meant "the hand", 6 "the new one", etc., and 10 was "two hands").

The most common system which arose, though, was base 10, clearly based on counting with both hands. This base obviously eventually dominated the rest. Some cultures worked with base 20 systems (in the case of the Aztecs and Mayans, a hybrid base 5−20 system), and a couple even used base 60: the Sumerians and Babylonians. For a global overview of traditional numerical bases, see Figure 3.

Each base has limitations. Small bases like 2 and 5 make counting to high numbers more difficult; 10 is not divisible by many numbers; and using 60 makes tables of numbers (for example, reciprocal tables) extremely large. In retrospect, base 8 or 12 would have been good choices, but clearly the development of our number systems by and large followed our biological configuration.

The manner in which number words are combined to make others is worth inspecting as well; different methods were used in many languages. One common approch is conglomeration, which English uses extensively; witness "one hundred twenty−five." A slightly more complex technique is the use of subtraction: the Yoruba of southwestern Africa call each of their numbers from 16 to 19 "20 less n." Another approach is the creation of new names for each new magnitude of number: witness the ancient Hindi setup, where specific, unrelated words existed for each power of 100 up to 10^{69}! Some cultures had different number words depending on the thing they were counting; see Figure 4 for an (extreme) example.

Written numerals

Several different approaches were taken to writing numerals; peoples' writing tools and styles varied, from scratching marks into clay, drawing on papyrus, and so on. The biggest division between styles is that of positional versus nonpositional systems; whether the (relative) location of a numeral changes its value. In positional systems such as our own, where the same numeral in different columns signify different magnitudes, algorithms for arithmetic are simple and consistent: for example, the way we know to go about long−division would work for any positional number system in any base.

Nonpositional systems make it much more difficult to perform arithmetic. Ancient Babylonians had to carry around huge tables of reciprocals in order to be able to divide, and Romans and Egyptians had to use complex multi−step processes to perform simple multiplication. The factor they lacked was the placeholder zero; without it, positional systems are impossible. It was invented in a handful of ancient cultures separately, but not assimilated into a mathematical canon until around the year 0.

The way in which these cultures came up with their written numerals is another issue.

Many simply ordered their alphabet and assigned values to each character: the Greeks, Hebrews, and Egyptians did this. The Mayans used complex illustrations of gods, which traditionally signified months, for their calculations. The Babylonians did what was convenient given their styli and clay tablets, using a simple system of wedged carvings. The "Arabic" numerals which we use today actually originated in India, and have clearly undergone much change. Some of the other, now unused, numeral sets are quite beautiful.

Arithmetic

We have seen that with positional number systems arithmetic is fairly easy. In the case of more primitive and cumbersome nonpositional systems, how were arithmetical operations performed? Addition and subtraction are slightly more complex, but with good attention to the numbers of symbols of one magnitude, it can be done without too much trouble. Multiplication and division, clearly, are more difficult. The most common approach to achieve those operations was repetitive addition or subtraction. Frequently the steps of the addition would be reduced logarithmically with a shortcut; a mathematician would calculate binary powers of his number, and add or substract certain values (e.g., $14 = 2^4 - 2^2$). The Babylonians, in contrast, would simply carry around canonical values of reciprocals so that they could divide (by the fact that $^x/_y = x \times {}^1/_y$). Considering the difficulty involved in performing such simple tasks, it is not suprising that these cultures didn’t develop much more advanced mathematics.

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